

Radially interrupted viscous fingers in a lifting Hele-Shaw cell

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Abstract. Viscous fingers have been produced in the lifting Hele-Shaw cell (LHSC), with concentric circular grooves etched onto the lower plate. The invading fluid (air) enters the defending newtonian fluid–olive oil, as fingers proceeding radially inwards towards the centre. The fingers are interrupted at the circular groove, and reform as secondary fingers. The effect of the grooves is to speed up the fingering process considerably and the fingers now reach the centre much faster. We explain this by comparing the variation with time, in velocity of the fingers in the normal LHSC. and the grooved cells. In the normal lifting HS cell the fingers move fastest on initial formation and slow down later. Since in case of the grooved plate, the fingers reform and receive a boost in their speed each time they encounter a groove, the fingers proceed to the centre faster.

PACS. 47.20.Gv Viscous instability – 47.54.+r Pattern selection; pattern formation – 68.03.-g Gas-liquid and vacuum-liquid interfaces

Viscous fingers (VF) are formed when a fluid of lower viscosity is forced into a fluid of higher viscosity [1,2]. The interface between the two fluids, develops finger-like intrusions which sometimes branch repeatedly into fractal patterns. Viscous fingering belongs to the well-known family of Laplacian growth phenomena which include for example–diffusion limited aggregation, dendritic growth and dielectric breakdown [3].

The process is studied conveniently in the Hele-Shaw (HS) cell, where the more viscous defending fluid is confined in a narrow gap between two glass plates. Characteristics of the pattern depend on viscosities of the fluids, interface tension between them and also details of the experimental arrangement and forcing pressure. In the conventional HS cell, the invading fluid enters the gap through a hole in the centre of the upper plate. In the Lifting Hele-Shaw cell (LHSC), the upper plate is slowly lifted, either from one end [4], or keeping it parallel to the lower plate, in this case the invading fluid enters from the sides forming a radial pattern [5]. The LHSC has not been studied as extensively as the conventional HS cell. It is useful for investigation of adhesion and is receiving a lot of attention at present [6,7]. Bohr et al. [8] have reported patterns in the lifting Hele-Shaw cell, using constant force to separate the plates.

The Hele-shaw cell can be modified in a number of ways to produce interesting variation in the patterns [9]. One way is to etch grooves onto the lower plate, this in-

roduces an anisotropy which affects the patterns [9], or stretch a piece of cloth across the lower plate [10]. It has also been shown that besides the symmetry of the superposed pattern the local geometry also plays a crucial role [11]. So far there has however, been no report of study of patterns in the LHSC with any such perturbations.

One expects LHSC patterns on etched plates to show interesting features too. The anisotropy imposed on the HS plate is usually a lattice with some definite symmetry, a single straight wire has also been used [12]. We want to investigate the effect of a perturbation which preserves the radial geometry of the set-up. A set of concentric grooves provides the requisite symmetry. The LHSC is intimately related with the problem of adhesion. A test for the effectiveness of a pressure sensitive adhesive (PSA) may be the time for which two plates separated by the adhesive can withstand the advance of viscous fingers which ultimately lead to debonding. So it is interesting to see the effect of perturbing the plates in different ways on the speed of advance of viscous fingers. These ideas motivate the present study.

We report the development of LHSC patterns with concentric circular grooved etched on the lower plate. The etched pattern is radially symmetric conforming to the LHSC arrangement and the fingers entering the cell from the sides are interrupted normally. This causes the fingering pattern to be disrupted and reformed each time it meets a groove. The appearance and width of the fingers change and the most surprising result is that the entire fingering pattern proceeds much faster and the fingers now

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Table 1. Fingering in LHSC with the lower plate having no groove (plane), one groove (1-grv) and three grooves (3-grv). Lifting force is different in the different sets. The initial diameter of the fluid blob (diam), the time for fingers to reach the centre (t_c), and the time of separation (t_s) are shown.

| set | type | vol (μl) | diam. (cm) | t_c (sec) | t_s (sec) |
|-----|-------|-----------------------|------------|-------------|-------------|
| I | plane | 150 | 5.2 | 4.6 | 7.9 |
| | 3-grv | 150 | 4.5 | 1.3 | 8.0 |
| II | plane | 150 | 5.7 | 28.5 | 34.5 |
| | 1-grv | 150 | 5.7 | 28.1 | 34.7 |
| | 3-grv | 150 | 5.5 | 0.9 | 32.9 |

reach the centre in a fraction of the time it takes without the grooves.

We plan to extend this study to non-Newtonian viscoplastic pastes and clays, which have properties closer to commercial adhesives, but in this preliminary work we use a newtonian fluid, where the physics of fingering is simpler.

Our lifting Hele-Shaw cell consists of two thick (~ 0.5 cm) glass plates. The lower one is fixed to a rigid frame, and the upper one can be lifted using a pneumatical cylinder arrangement. The lifting force can be adjusted. In the standard LHSC for studying adhesion the velocity of plate separation is constant, whereas in our apparatus the lifting force is controlled and kept constant during the experiment, allowing the velocity to vary. The process of pattern formation is recorded by a CCD camera (at the rate of 25 frames per second) and analysed using image-pro plus software. The defending fluid is placed on the lower plate, and the upper plate pressed down on it. This makes the fluid form a circular blob with diameter of several cm., the upper plate is then lifted slowly.

In the present modification we use two different lower plates, one has a single circular groove (diameter 19 mm) etched on it, and the other has three concentric grooves (diameter 9 mm, 17.5 mm and 24.5 mm). The width of the grooves is about 0.1 cm, and depth ~ 0.05 cm. The patterns are compared with normal LHSC with a plane lower plate. The defending fluid used here is olive oil (coloured slightly with a dye), this is a newtonian fluid and the invading fluid is air (assumed to have zero viscosity).

Figures 1a–b show two successive stages in pattern formation, when the lower plate has no grooves. Figures 1c–d and Figures 1e–f show respectively the patterns with one and three grooves. Table 1 shows the results of measurements—we take a measured volume of olive-oil using a micro-pipette. Different sets are taken with different force of separation, in the present arrangement though the force can be varied, we cannot measure it quantitatively. The experiments in set-I are all done at the same lifting force, which is higher than the constant lifting force used in set-II.

In Table 1, the diameter of the initial circular pattern before fingering starts is given, t_c is the time required for the fingers to reach the centre of the pattern and t_s , is the time required for complete separation of the plates. Comparison of t_s for sets I and II show that the lifting force is much less in set II. The results for t_c show that

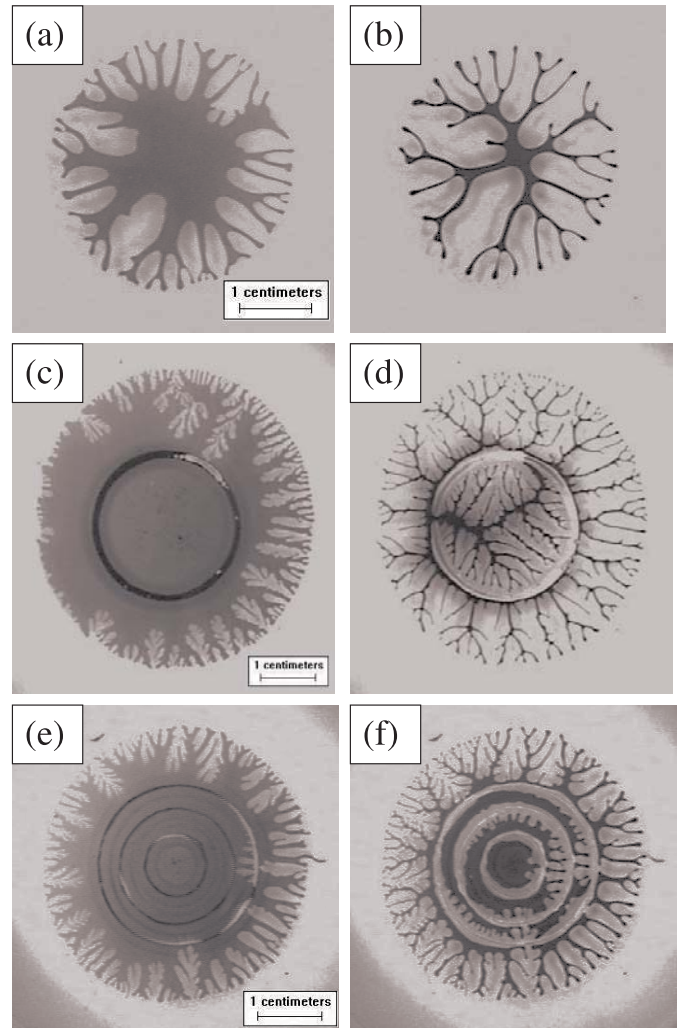


Fig. 1. (a) and (b) show an early and final stage of patterns with plane lower plate, (c) and (d) are for the single-grooved plate and (e) and (f) for the three-grooved plate.

for the 3-grooved plate, the time for fingers to reach the centre is always much smaller than for the plane plate. For the single grooved plate, the effect is not so noticeable. The time required for the plates to separate completely is however, independent of the presence of grooves, within the accuracy of this experiment (see Tab. 1). This is expected, since the total energy required to separate the plates should depend on the radius of the initial blob and its thickness only.

Each result shown in the table is an average over 4–7 patterns. The fingering is a stochastic process and the patterns are not perfectly symmetric, so the fingering times reported are not exact quantities, but the gross characteristics can be identified in spite of a certain amount of uncertainty.

To explain the difference in t_c between patterns with and without grooves, we look at the finger velocities in the two cases. Figure 2 shows how the velocity of the fastest finger varies with time. In Figure 2a we show velocity vs. time for a typical pattern without grooves. The velocity is

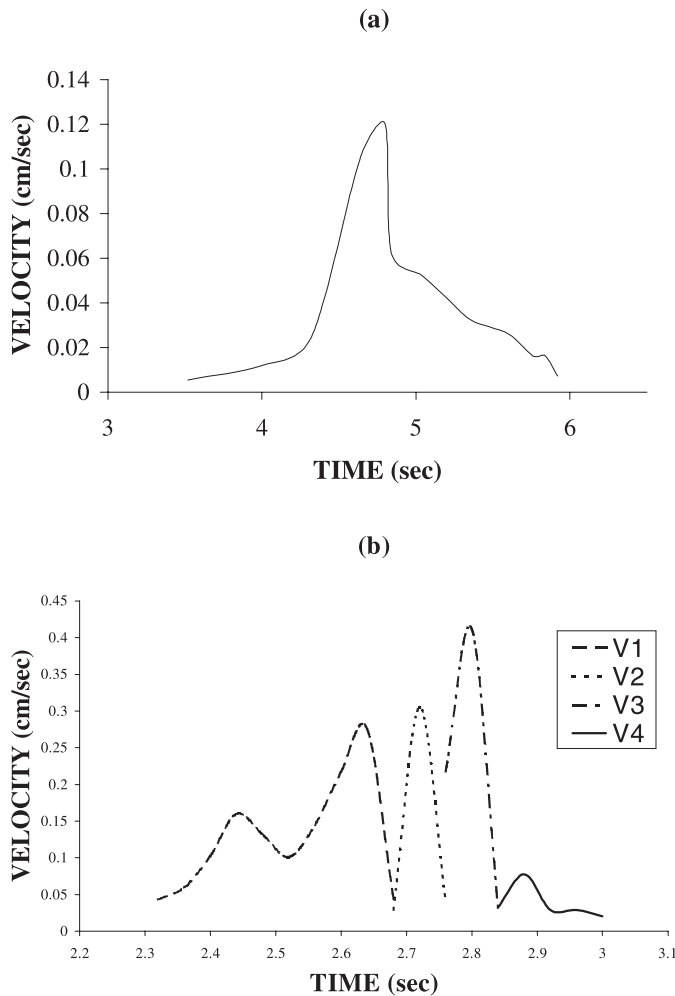


Fig. 2. (a) and (b) show velocity of the fastest finger vs. time for the plane and three-grooved plate respectively. In (b) v_1 is before reaching the largest groove, v_2 between largest and middle groove, v_3 between middle and smallest groove and v_4 from smallest groove to centre.

seen to be highest at the outset, it later undergoes some fluctuations but on the whole decreases, ultimately going to zero. In the grooved plate patterns we observe that as soon as one or two of the competing fingers reach the largest groove the air spreads very rapidly around the groove, and then starts forming secondary fingers from the groove, which look exactly like the initial fingers. The secondary fingers again start with the very high initial velocity of the primary fingers. This process is repeated at the next groove. So on the whole the fingers proceed towards the centre with a much higher average velocity compared to the plane plate, making t_c much smaller. For the single-grooved plate, the lifting force in set-II is much smaller, so the fingers do not reach the point where they speed up within the groove. We are at present, studying the effect of different lifting forces more thoroughly.

Besides the gross difference observed in t_c , a close look at Figure 1, reveals striking differences in the fingers of Figure 1a with Figures 1c and e, even before the air en-

counters the first groove. In Figures 1c and e the fingers are much narrower compared to Figure 1a, though the lifting force is the same for all these patterns. Further, the competition for space between the fingers, which leads to reduction in the number of growing fingers is much stronger in Figures 1a–b. This implies that, the pressure distribution within the oil-blob is strongly affected by the presence of the groove. The groove imposes a circular ring of constant pressure, which reduces the instability in the system. In an ideal system, even without grooves we would expect the isobars to be perfect concentric circles, but non-uniformities in the plate and other fluctuations cause deviations. Since the groove acts as a perfectly circular isobar, pressure gradients at adjacent fingers in the grooved experiment are similar and the fingers grow together, with less competition.

In the grooved plates there is a strong tendency towards cavitation. It is quite difficult to eliminate air-bubbles completely and it is possible that minute quantities of air remain trapped in the grooves or in irregularities in the plates, these are initially too small to be seen by the naked eye, but start expanding as the upper plate is lifted. Cavitation and its role in adhesion has been discussed previously by Gay et al. [7]. The initial width and velocity of fingers has been studied theoretically by linear stability analysis [13], but the development of finger velocities up to breakthrough or up to complete separation of the plates in LHSC has not been studied in detail. Shelley et al. [14] have studied the full formation of the LHSC pattern numerically, their results indicate that in a newtonian fluid the finger velocity first increases, but then drops off as the fingers approach the centre of the plate. This is similar to what we observe with the newtonian fluid-olive-oil.

In context of the adhesion problem, our present results indicate that presence of grooves will reduce effective adhesion, at least for Newtonian fluids. However, substances commonly used as adhesives are highly non-newtonian, and the effect of grooves or similar perturbations may be quite different in this case. Earlier work with a non-newtonian fluid showed that the variation of finger velocity in this case does not resemble the curve in Figure 2a. The 5–6 dominant fingers in this case, which finally reach the centre of the pattern, actually speed up towards the end of the process [15]. So there is a need for further work in this direction.

In conclusion we emphasize that several factors may lead to interesting variations in the patterns with circular grooves – rheological properties of the fluids, newtonian and non-newtonian character as well as the number, spacing and depth of the grooves may play a crucial role. We hope to report further results in the near future.

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